



RIMS 共同研究（公開型）「保型形式の研究」  
RIMS conference “Research on automorphic forms”

代表：青木 宏樹（東京理科大学）      Organizers: Hiroki Aoki (Tokyo University of Science)  
副代表：並川 健一（東京電機大学）      Kenichi Namikawa (Tokyo Denki University)

日程： 2024 年 1 月 22 日（月）午後 – 26 日（金）午前  
場所： 京都大学数理解析研究所 420 号室

Date: January 22rd (Mon.) P.M. – January 26th (Fri.) A.M. 2024  
Venue: Room 420, Research Institute for Mathematical Sciences (RIMS),  
Kyoto University, Kyoto 606-8502, Japan

講演概要      Abstract

**1 月 22 日（月）      Jan. 22 (Mon.)      12:45 – 13:45**

比嘉 陸（東京理科大学）      Riku Higa (Tokyo University of Science)

「奇数次 Galois 拡大体上の既約ルート格子と符号理論」

“Galois trace forms of type  $A_n$ ,  $D_n$ ,  $E_n$  for odd  $n$  and codes”

有理数体上の奇数次ガロア拡大体に埋め込める既約ルート格子を決定する。また、そのルート格子と符号の対応から偶ユニモジュラー格子を構成する方法について報告する。

We determine the irreducible root lattices that can be embedded in a Galois extension over  $\mathbb{Q}$  of odd degree. Furthermore, I report a method for constructing even unimodular lattices from these lattices and the correspondence with codes.

**1 月 22 日（月）      Jan. 22 (Mon.)      13:45 – 14:45**

Yiyang Wang（京都大学）      Yiyang Wang (Kyoto University)

“Formal degrees and parabolic induction: the maximal case”

The formal degree conjecture of Hiraga–Ichino–Ikeda expresses the formal degree of a discrete series representation of a reductive  $p$ -adic group in terms of its adjoint gamma factor and Langlands parameter. For a non-supercuspidal discrete series representation, it’s natural to study the relation of its formal degree to the formal degree of its cuspidal support. In this talk, I’ll explain how to use tools from local harmonic analysis to deal with the case of a maximal parabolic subgroup, together with some potential applications.

1 月 22 日 (月)      Jan. 22 (Mon.)      15:00 – 16:00

齋藤 陽平 (慶應義塾大学)      Yohei Saito (Keio University)

**“Theta lift to certain cohomological representations of indefinite orthogonal groups”**

Howe and Tan investigated a degenerate principal series representation of indefinite orthogonal groups and completely described its composition series. Their work implies the existence of a unique unitarizable irreducible submodule, which is isomorphic to a non-tempered derived functor module. Our research focuses on the theta liftings locally corresponding to this irreducible representation. In this talk, we introduce a special choice of Schwartz functions to define the liftings, enabling us to provide precise descriptions of the Fourier expansions of resulting automorphic forms using Borchers’ method. This is joint work with Takuya Miyazaki.

1 月 22 日 (月)      Jan. 22 (Mon.)      16:00 – 17:00

小原 和馬 (東京大学)      Kazuma Ohara (The University of Tokyo)

**“Types for Bernstein blocks and their Hecke algebras”**

The category of smooth complex representations of  $p$ -adic groups decomposes into the product of full subcategories, called Bernstein blocks. In this talk, I will explain the result that (under a mild condition,) every block is equivalent to a depth-zero block, that is much better understood than general blocks. This result is obtained by using the theory of types and an isomorphism of Hecke algebras. This is a joint work with Jeffrey Adler, Jessica Fintzen, and Manish Mishra.

1 月 23 日 (火)      Jan. 23 (Tue.)      9:30 – 10:30

伊藤/Sprung Florian (アリゾナ州立大学)      Florian Ito/Sprung (Arizona State University)

「Bloch–加藤 Selmer 群と奇偶 Selmer 群の関係について」

**“On relationships between Bloch–Kato Selmer groups and signed Selmer groups”**

保型形式に伴う (Bloch–加藤) Selmer 群が円分  $\mathbb{Z}_p$ -拡大などを通して研究されているが、素数  $p$  が non-ordinary な場合、Selmer 群の  $\mathbb{Z}_p$ -拡大での振る舞いが複雑である。小林真一氏がより扱いやすい「奇偶 Selmer 群」を構成しはじめたが、Bloch–加藤 Selmer 群との正確な関係は現在未解決である。Selmer 群が有限である仮定のもとで奇偶 Selmer 群とのある関係を紹介したいと思います。(J.Ray 氏との共同研究)

The (Bloch–Kato) Selmer group attached to a modular form can be studied by considering it in cyclotomic  $\mathbb{Z}_p$ -extensions, but for non-ordinary primes  $p$ , its behavior along these  $\mathbb{Z}_p$ -extensions can be quite complex. S. Kobayashi formulated analogous “signed” (or “chromatic”) Selmer groups which are easier to handle, but its precise relation to the Selmer groups is not entirely clear. We would like to present recent works with J. Ray, in which we assume the Selmer group is finite and derive some relation with the chromatic Selmer group, among other results.

1 月 23 日 (火)      Jan. 23 (Tue.)      10:45 – 11:45

村瀬 篤 (京都産業大学)      Atsushi Murase (Kyoto Sangyo University)

成田 宏秋 (早稲田大学)      Hiro-aki Narita (Waseda University)

**“On Arakawa lifting”**

We give a criterion for a Hecke eigenform on the quaternion unitary group of degree two to be an Arakawa lift in terms of a special value of an automorphic L-function.

**1月23日 (火)      Jan. 23 (Tue.)      13:15 – 14:15**

**Nils-Peter Skoruppa (ジューゲン大学)      Nils-Peter Skoruppa (University of Siegen)**

**“Dessins d’enfants, network flows and modular forms”**

Modular forms can be efficiently computed using the theory of Manin symbols. We propose a related theory using the language of graphs and the theory of network flows. This theory does not only provide faster computation of modular forms but also new theoretical insight since it turns out that we can naturally extend this theory to arbitrary dessin d’enfants.

**1月23日 (火)      Jan. 23 (Tue.)      14:30 – 15:30**

**Siegfried Böcherer (マンハイム大学)      Siegfried Böcherer (University of Mannheim)**

**“ $p$ -adic limits of Siegel Eisenstein series (joint work with T. Kikuta)”**

Recently we developed a structure theory of  $\text{mod } p^m$  singular holomorphic Siegel modular forms. We use this as a tool in investigating the  $p$ -adic limits of Siegel Eisenstein series (level one). Previous works (Nagaoka, Katsurada and others) already showed in some cases that such  $p$ -adic limits may become classical modular forms of level  $\Gamma_0(p)$ . Our method is simpler and covers more general cases; we also get a natural explanation, why such limits become linear combinations of certain genus theta series.

The starting point is the observation that Siegel Eisenstein series of certain weights are actually  $\text{mod } p^m$  singular, i.e. all Fourier coefficients of maximal rank are congruent zero  $\text{mod } p^m$ .

**1月23日 (火)      Jan. 23 (Tue.)      15:45 – 16:45**

**伊原 康隆 (京都大学)      Yasutaka Ihara (Kyoto University)**

**“On Mikio Sato’s works related to automorphic forms – history and later developments –”**

Professor Mikio Sato (1928/4/18 ~ 2023/1/09) had strong interest also in number theory, including that of automorphic forms, especially on (i) the Ramanujan-Petersson conjecture and on (ii) what is later called the Sato-Tate conjecture. Sato’s break-through works on these subjects were done within two years (1961 ~ 63). Although unpublished, they strongly motivated later works by others which finally lead to an affirmative solution of each of these conjectures. And because unpublished, some of us, including myself who was then a graduate student, feel obliged to pass the essence of what they have learned from Sato to the next generation. This is my first motivation for having raised hand to be a pinch-hitter. After some hesitations I decided to focus mainly on (ii), because my interest on (ii) is much more than historical.

My second motivation (also added in this abstract to draw attention of non-participant readers) is related to a circumstance that the corresponding written versions are still sleeping. They are the two commentaries on (i) and (ii), asked for and written by myself, and accepted for publication in 2020. Here, by publication, I mean a planned Volume of Collected Works of Mikio Sato by a world-wide publisher. A characteristic feature of Sato’s case is that he left several important works unpublished and hence the corresponding commentaries were asked for this Volume and were contributed. I should add that the number-theory related is just a small part. Until publication, these commentaries are not directly accessible. This is different from the usual Collected Papers. I strongly hope that this Volume containing such commentaries as a characteristic part, be published in time. I feel that a generation-gap can be fatal for the Volume to remain vivid as a legend.

**1 月 24 日 (水)      Jan. 24 (Wed.)      9:30 – 10:30**

**村上 友哉 (九州大学)      Yuya Murakami (Kyushu University)**

**「種々の量子不変量と偽テータ関数の radial limits」**

**“Various quantum invariants and radial limits of false theta functions”**

Quantum invariants are important topological objects motivated by physics. They are also important in number theory since they relates to modular forms. Such relations are formulated as “radial limit conjectures” in some cases. In this talk, I prove such conjectures for various quantum invariants.

**1 月 24 日 (水)      Jan. 24 (Wed.)      10:45 – 11:45**

**Brandon Williams (アーヘン工科大学)      Brandon Williams (RWTH Aachen)**

**“Modular forms with poles on hyperplane arrangements”**

I will describe a family of hyperplane arrangements in lattices of signature  $(n, 2)$  for which the graded rings of modular forms with poles on those hyperplanes are freely generated. The largest example is a ring of modular forms for the lattice  $2U \oplus D_{11}$ , which is a polynomial algebra on 14 generators. This is joint work with Haowu Wang.

**1 月 24 日 (水)      Jan. 24 (Wed.)      13:15 – 14:15**

**山内 卓也 (東北大学)      Takuya Yamauchi (Tohoku University)**

**「超特殊アーベル多様体上の同種グラフ：固有値, Bruhat–Tits ビルディング および Property (T)」**

**“Isogeny graphs on superspecial abelian varieties: Eigenvalues, Connection to Bruhat–Tits buildings, and Property (T)”**

正整数  $g$ , 素数  $p, \ell$  に対して, 標数  $p$  の有限体上の次元  $g$  をもつ superspecial abelian varieties であって  $\ell$ -marking が指定されたものの全体の成す類から有限向き付き正則グラフを構成することができる. 講演ではこのグラフに対するランダムウォーク行列の性質を対応する Bruhat–Tits buildings を解析することで調べることができることを説明する. また, 保型形式, 保型表現論の観点からランダムウォーク行列の固有値に関して何が期待されるかも説明する. 本研究は東京大学の相川勇輔氏、京都大学の田中亮吉氏との共同研究である.

For each fixed integer  $g \geq 2$ , a prime  $p$ , and all primes  $\ell$  with  $\ell \neq p$ , we can consider finite regular directed graphs associated with the set of equivalence classes of  $\ell$ -marked principally polarized superspecial abelian varieties of dimension  $g$  in characteristic  $p$ . In this talk, I will explain that we can study such graphs in terms of the corresponding Bruhat–Tits buildings. I also discuss the eigenvalues values of the random walk matrices in view of the theory of automorphic representations. This is a joint work with Y. Aikawa (Tokyo university) and R. Tanaka (Kyoto university).

**1 月 24 日 (水)      Jan. 24 (Wed.)      14:30 – 15:30**

**伊吹山 知義 (大阪大学)      Tomoyoshi Ibukiyama (Osaka University)**

**“Dimensions of paramodular forms with involution”**

Paramodular forms are interesting object, related to generalized Shimura–Taniyama conjecture, supersingular abelian varieties, and new form theory. We give explicit dimension formulas of paramodular forms of prime level with plus and minus signs of the Atkin–Lehner involution as well as those of corresponging algebraic modular forms.

**1 月 24 日 (水)      Jan. 24 (Wed.)      15:45 – 16:45**

成田 宏秋 (早稲田大学)      **Hiro-aki Narita** (Waseda University)

**“Fourier-Jacobi expansion of cusp forms generating quaternionic discrete series”**

Aaron Pollack provided a theory of Fourier expansion along the Heisenberg parabolic subgroup for automorphic forms generating quaternionic discrete series. His Fourier expansion deals with some general class of simple groups but describe only the terms contributed by characters. However, to complete the Fourier expansion, which we want to call the Fourier-Jacobi expansion, we have to study the terms contributed by irreducible unitary representations called Stone von-Neumann representation. The aim of this talk is to report a recent progress toward the completion of the Fourier-Jacobi expansion.

**1 月 24 日 (水)      Jan. 24 (Wed.)      16:45 – 17:00**

「来年度の RIMS 研究集会について」

**“On the RIMS workshop next academic year”**

Announcement of the next RIMS conference and related workshops.

**1 月 24 日 (水)      Jan. 24 (Wed.)      18:00 – (Reception 17:40 –)**

懇親会      **Conference Dinner**

Advance registration is required. The venue is Camphora at the main campus of Kyoto University.

**1 月 25 日 (木)      Jan. 25 (Thu.)      9:30 – 10:30**

伊藤 望 (国立台湾大学)      **Nozomi Ito** (National Taiwan University)

「宮脇リフトのノルム公式について」

**“On the formula of the norms of Miyawaki lifts”**

宮脇リフトは池田氏の 2006 年の論文で定義された、池田リフトの対角制限を核関数に用いるリフティングである。池田氏は同論文で、宮脇リフトのノルムが満たす公式を予想した。本講演では、その公式が精密化 Gan–Gross–Prasad 予想に類似した公式に書き直せることを紹介する。

**1 月 25 日 (木)      Jan. 25 (Thu.)      10:45 – 11:45**

若槻聡 (金沢大学)      **Satoshi Wakatsuki** (Kanazawa University)

「 $GL_n$  の自己双対および共役自己双対表現の捻った跡の漸近挙動」

**“Asymptotic behavior for twisted traces of self-dual and conjugate self-dual representations of  $GL_n$ ”**

講演では主合同部分群のレベルに対する  $GL_n$  の自己双対および共役自己双対な離散保型表現の捻った跡の和の漸近挙動について議論する。いくつかの条件の下、捻った跡公式を用いることで、その漸近挙動の主要項を決定する。さらに共役自己双対表現の場合にはその漸近公式から保型密度定理を導く。この研究は高梨悠吾氏との共同研究です。

1月25日(木)      Jan. 25 (Thu.)      13:15 – 14:15

万 忱 (ラトガース大学)      **Chen Wan** (Rutgers University)

**“A local twisted trace formula for Whittaker induction of coregular symmetric pairs”**

In this talk, I will discuss the geometric expansion of a local twisted trace formula for the Whittaker induction of any symmetric pairs that are coregular. This generalizes the local (twisted) trace formula for reductive groups proved by Arthur and Waldspurger. As a consequence of the trace formula, we prove a simple local trace formula of those models for strongly cuspidal test functions which implies a multiplicity formula for these models. I will also present various applications of the trace formula and multiplicity formula, including a necessary condition for a discrete L-packet to contain a representation with a unitary Shalika model (resp. a Galois model for classical groups) in terms of the associated Langlands parameter, and we also compute the summation of the corresponding multiplicities for certain discrete L-packets. This is a joint work with Raphael Beuzart-Plessis.

1月25日(木)      Jan. 25 (Thu.)      14:30 – 15:30

小松 尚夫 (浙江理工大学)      **Takao Komatsu** (Zhejiang Sci-Tech University)

**“Diophantine problems of the equations  $x^2 - y^2 = z^r$ ”**

Similarly to the Pythagorean triple  $(s^2 - t^2, 2st, s^2 + t^2)$ , the equation  $x^2 - y^2 = z^r$  has the parameterizations for  $r \geq 2$ . By using them, we could give the convergents of certain values by rationals  $x/y$  such that  $x, y$  satisfy Diophantine equations. We can also give the Frobenius numbers formed from Diophantine equations. In this talk we consider the equations  $x^2 - y^2 = z^r$  and some related ones.

1月26日(金)      Jan. 26 (Fri.)      9:30 – 10:30

小嶋 久祉 (埼玉大学)      **Hisashi Kojima** (Saitama University)

**“On the representability of Hilbert cusp forms by theta series”**

The basis problem of modular forms is a fundamental and interesting theme. Applying trace formulas of Hecke operators, M. Eichler first provided a positive solution of the basis problem of elliptic cusp forms by theta series associated with quadratic forms. This is generalized by Hijikata, Pizer, Shemanske, Jacque-Langlands, Shimizu et al. J-L.Waldspurger established that elliptic cusp forms of arbitrary level are spanned by theta series using Zagier’s theorems and Siegel main theorem of quadratic forms. We generalize Waldspurger’s theorem and Zagier’s theorem to the case of Hilbert cusp forms of arbitrary level. As an application of the pullback formula of Eisenstein series, Bocherer solved the basis problem of Siegel modular forms of level one. Under some technical requirements concerning level and weight of modular forms, Kuang proved that Hilbert-Siegel cusp forms are spanned by theta series using representation theory. Our result is a joint work with Hiroshi Sakata.

1月26日(金)      Jan. 26 (Fri.)      10:45 – 11:45

周 海港 (同济大学)      Haigang Zhou (Tongji University)

**“The classification and representations of positive definite ternary quadratic forms of level  $4N$ ”**

Classifications and representations are two main topics in the theory of quadratic forms. In this talk, we consider these topics of ternary quadratic forms. For a given squarefree integer  $N$ , firstly we give the classification of positive definite ternary quadratic forms of level  $4N$  explicitly. Secondly, we give the weighted sum of representations over each class in every genus of ternary quadratic forms of level  $4N$  by using quaternion algebras and Jacobi forms. The formulas are involved with modified Hurwitz class number. As a corollary, we get a formula for the class number of ternary quadratic forms of level  $4N$ . As applications, we give an explicit base of Eisenstein series space of modular forms of weight  $3/2$  of level  $4N$ , and give new proofs of some interesting identities involving representation number of ternary quadratic forms.