RIMS Satellite Seminar
"Japan-Taiwan Number Therory Conference"

Organizers: Kenichi Namikawa (Tokyo Denki University)
Ming-Lun Hsieh (National Taiwan University)

Date: September 1st (Mon.) – September 5th (Fri.), 2025

Venue: Kyukamura Shikanoshima, 1803-1 Katsuma, Higashi-Ku, Fukuoka, Japan

Sep. 1 (Mon.)

14:00 – 14:50 **Julie Wang** (Academia Sinica) ♢

The GCD method

15:10 – 16:00 **Ting-Yu Lee** (National Taiwan University) ♢

Isometries of lattices with prescribed characteristic polynomials and Brauer–Manin obstructions

16:20 – 17:10 **Kaoru Sano** (Institute for Fundamental Mathematics in NTT) ♢

Arithmetic problems on parabolic parameters for quadratic polynomial maps

Sep. 2 (Tue.)

9:20 – 10:10 **Kazuki Morimoto** (Kobe University) ♠

On Whittaker periods for orthogonal groups and symplectic groups

10:30 – 11:20 **Yiyang Wang** (Kyoto University) ♢, ♠

Periods of Eisenstein series: the case $\mathrm{GL}_n \times \mathrm{GL}_m \backslash \mathrm{GL}_{n+m}$, $n > m$

11:40 – 12:30 **Hsin-Yen Ho** (National Taiwan University) ♢

Bessel periods of Klingen–Eisenstein series

14:00 – 14:50 **Kengo Fukunaga** (Institute of Science Tokyo) ♠

Multi-variable p-adic power series of logarithmic order and multi-variable admissible distributions

15:10 – 16:00 **Chi-Yun Hsu** (Santa Clara University) ♢

p-adic companion forms for Yoshida lifts

16:20 – 17:10 **Naoto Dainobu** (Kyushu University) ♠

On the ideal class groups of the division fields of elliptic curves

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♠:White board. ♢: Projector.

This conference is supported by RIMS.

Sep. 3 (Wed.)

9:20 – 10:10 **Hung-Chun Tsui** (National Tsing Hua University) ♢

On q-shuffle Relations for Multiple Eisenstein Series in Positive Characteristic

10:30 – 11:20 **Peng-Jie Wong** (National Sun Yat-Sen University) ♢, ♠

Distributions of central values and orders of Sha groups in families of quadratic twists

11:40 – 12:30 **Tomo Narahara** (Osaka Metropolitan University) ♢, ♠

An elementary proof of the Berndt–Arakawa formula

14:00- free discussion

Sep. 4 (Thu.)

9:20 – 10:10 **Miyu Suzuki** (Kyoto University) ♠

Distinguished discrete series representations

10:30 – 11:20 **Yao Cheng** (Tamkang University)

TBA

11:40 – 12:30 **Nozomi Ito** (National Taiwan University) ♢

Between $\mathfrak{z}$-finiteness and Hecke-finiteness

14:00 – 14:50 **Shu-Yen Pan** (National Tsing Hua University) ♢

On the Classification of Irreducible Representations of Finite Classical Groups

15:10 – 16:00 **Hiroshi Ishimoto** (Osaka Metropolitan University) ♢

Arthur's classification for non-quasi-split odd special orthogonal groups

16:20 – 17:10 **Masao Oi** (National Taiwan University) ♠

Green functions for positive-depth Deligne–Lusztig induction

Sep. 5 (Fri.)

9:20 – 10:10 **Satoshi Kumabe** (Kyushu University) ♢

Zeta functions of certain two-parameter families of K3 surfaces via the Appell functions

10:30 – 11:20 **Ryota Shii** (Kyushu University) ♢

On the Mazur–Tate refined conjecture for the anticyclotomic tower at inert primes

11:40 – 12:30 **Hohto Bekki** (Saga University) ♠

Regulators and L-values of some Fermat hypersurfaces

Abstract

Sep. 1 (Mon.)

14:00 – 14:50 **Julie Wang** (Academia Sinica)

Title: The GCD method

Abstract: The GCD method, introduced by Corvaja and Zannier, was initially developed in the function field setting to study integral points on $\mathbb{P}^2 \setminus D$, where D is the union of a smooth conic and two lines in general position. With the foundational GCD theorem later established by Aaron Levin in the number field case, the method has since been extended and applied in significantly broader contexts.

Recently, the method has emerged as a powerful tool in Diophantine geometry over function fields. In joint work with Guo, Nguyen, and Sun, it has been applied to Vojta's generalized *abc* conjecture for algebraic tori, as well as to the Lang–Vojta Conjecture for varieties of log general type that arise as ramified covers of tori over function fields.

A complex analytic counterpart of the method has also been developed. In collaboration with Min Ru, we have applied it to cases of Campana's orbifold conjecture, particularly for toric varieties that are highly ramified along their boundaries.

In this talk, I will survey the development of the GCD method and report on its recent progress and applications in both the function field and complex analytic settings.

15:10 – 16:00 **Ting-Yu Lee** (National Taiwan University)

Title: Isometries of lattices with prescribed characteristic polynomials and Brauer–Manin obstructions

Abstract: Given a reciprocal polynomial f and a lattice L, one asks if there is an isometry of L with characteristic polynomial f. In a series of papers of Eva Bayer–Fluckiger and others, they give sufficient and necessary conditions for such an isometry to exist. In this talk, I will approach this problem from a different point of view. I will first construct a homogeneous space of SL_n, and show that each rational point of the homogeneous space corresponds to an isometry of the vector space associated to L. In this setting, the isometries of the lattice itself corresponds to integral points of the homogeneous space. Then we use the integral Brauer–Manin obstructions and strong approximation to solve this problem. This is a joint work in progress with Y. Cao, Y. Hu and F. Xu.

16:20 – 17:10 **Kaoru Sano** (Institute for Fundamental Mathematics in NTT)

Title: Arithmetic problems on parabolic parameters for quadratic polynomial maps

Abstract: The parabolic parameters play an important role in the complex dynamics. In 1995, Morton and Vivaldi defined the polynomials whose roots are parabolic parameters for a one-parameter family of polynomial maps. We call these polynomials delta factors. Since the parabolic parameters involve the cyclotomic field, they are not only complex dynamical objects but also number theoretic objects. Morton and Vivaldi conjectured that the delta factors are irreducible over the rational field. However, it is still not known in general. Recently, we proved the irreducibility of the delta factors of period 3 for quadratic polynomials. In this talk, we introduce the parabolic parameters and some arithmetic problems arising around the parabolic parameters. This is a joint work with Junnosuke Koizumi, Yuya Murakami, and Kohei Takehira.

Sep. 2 (Tue.)

9:20 – 10:10 **Kazuki Morimoto** (Kobe University)

Title: On Whittaker periods for orthogonal groups and symplectic groups

Abstract: Lapid and Mao formulated a conjecture on Ichino-Ikeda type formula of Whittaker periods for quasi-split reductive groups and metaplectic groups. Moreover, in the case of metaplectic groups and unitary groups, they conjectured certain local identities and showed that their global conjecture follows from this local conjecture. In this talk, we formulate a similar local conjecture and show that their global conjecture follows from our local conjecture in the case of orthogonal groups and symplectic groups. This talk is based on a joint work with Erez Lapid and Zhengyu Mao.

10:30 – 11:20 **Yiyang Wang** (Kyoto University)

Title: Periods of Eisenstein series: the case $\mathrm{GL}_n \times \mathrm{GL}_m \backslash \mathrm{GL}_{n+m}$, $n > m$

Abstract: I shall briefly explain the framework and conjectures of Sakellaridis-Venkatesh for the example $\mathrm{GL}_n \times \mathrm{GL}_m \backslash \mathrm{GL}_{n+m}$, $n > m$. Though geometrically this is a well-understood symmetric space, there are some new analytic phenomena for this example. It shows that Zydor's truncation is not quite enough for very general periods, and one needs another regularization in this case (in progress).

11:40 – 12:30 **Hsin-Yen Ho** (National Taiwan University)

Title: Bessel periods of Klingen–Eisenstein series

Abstract: In this talk, we give an explicit formula of Bessel periods of Klingen-Eisenstein series on GSp(4) and explain the applications to the non-vanishing modulo p of Eisenstein series.

14:00 – 14:50 **Kengo Fukunaga** (Institute of Science Tokyo)

Title: Multi-variable p-adic power series of logarithmic order and multi-variable admissible distributions

Abstract: Amice–Velu (1975) and Vishik (1976) introduced one-variable p-adic power series of logarithmic order and admissible distributions. Further, they constructed one-variable p-adic L-functions attached to non-ordinary cusp forms.

In this talk, we introduce multi-variable generalizations of the notations defined by Amice–Velu and Vishik. In addition, we present our results which I believe are essential for the theory of multi-variable p-adic L-functions attached to non-ordinary cusp forms.

This is a joint work with Tadashi Ochiai (Institute of Science Tokyo).

15:10 – 16:00 **Chi-Yun Hsu** (Santa Clara University)

Title: p-adic companion forms for Yoshida lifts

Abstract: Coleman showed that the $(k-1)$st power of the theta operator $q\frac{d}{dq}$ defines a map from overconvergent forms of weight $2-k$ and slope 0 to weight k and slope $k-1$. Moreover, the critical p-stabilization of a classical CM form is the image of a p-adic CM form, strengthening the fact that its Galois representation splits locally at p. In the GSp_4 setting, the Galois representation of a Yoshida lift splits locally into two 2-by-2 blocks at p. In joint work in progress with Bharathwaj Palvannan, we aim to prove an analogous strengthening. The relevant theta operator arises from the last differential of the dual BGG complex. We computed its explicit effect on q-expansions for weight $(k,3)$, and expect that the effect for general weights to be a power of this. Using the explicit Fourier coefficients of Yoshida lifts by Hsieh–Namikawa, we show that Yoshida lifts lie in the image of this theta operator.

16:30 – 17:20 **Naoto Dainobu** (Kyushu University)

Title: On the ideal class groups of the division fields of elliptic curves

Abstract: For an elliptic curve E over $\mathbb{Q}$ and a prime number p, we consider the p-th division field $K := \mathbb{Q}(E[p])$ of E. This gives a $\mathrm{GL}_2(\mathbb{F}_p)$-extension over $\mathbb{Q}$ in most cases. In this talk, we provide some new results on the structure of the ideal class group $\mathrm{Cl}(K)$ of K as a $\mathrm{Gal}(K/\mathbb{Q})$-module via the fine Selmer group of E. These results give partial refinements of recent works by Prasad–Shekhar and Hiranouchi–Ohshita. We also discuss a relationship between the ideal class group $\mathrm{Cl}(K)$ and L-functions associated with E, which can be regarded as an analogue of the classical result by Herbrand and Ribet.

Sep. 3 (Wed.)

9:20 – 10:10 **Hung-Chun Tsui** (National Tsing Hua University)

Title: On q-shuffle Relations for Multiple Eisenstein Series in Positive Characteristic

Abstract: Thakur defined multiple zeta values (MZVs) in positive characteristic and showed that the product of two such values can be expressed as an $\mathbb{F}_p$-linear combination of MZVs of the same weight. These are now called *q-shuffle relations*. In this talk, I will introduce multiple Eisenstein series in positive characteristic, which have expansions involving "*multiple Goss sums*", with Thakur's MZVs appearing as the "*constant terms*". I will then discuss how the q-shuffle relations for MZVs can be lifted to those for multiple Eisenstein series.

10:30 – 11:20 **Peng-Jie Wong** (National Sun Yat-Sen University)

Title: Distributions of central values and orders of Sha groups in families of quadratic twists

Abstract: As a refinement of Goldfeld's conjecture, there is a conjecture of Keating-Snaith asserting that $\log L(\frac{1}{2}, E_d)$, over certain quadratic twists E_d of an elliptic curve $E/\mathbb{Q}$, behaves like a normal random variable. In light of this, Radziwiłł and Soundararajan conjectured that the distribution of $\log(|\mathrm{Sha}(E_d)|/\sqrt{|d|})$ is also approximately Gaussian for those E_d, and proved that the conjectures of Keating-Snaith and theirs are both valid "from above". More recently, under GRH, they further established a lower bound for the involved distribution towards Keating–Snaith's conjecture.

In this talk, we will discuss some extensions of the above-mentioned conjectures and how to adapt Radziwiłł-Soundararajan's method to obtain lower bounds of the expected distributions under GRH. If time allows, as an application, we will discuss a simultaneous non-vanishing result for central L'-values in families of quadratic twists of triples of holomorphic modular forms.

11:40 – 12:30 **Tomo Narahara** (Osaka Metropolitan University)

Title: An elementary proof of the Berndt–Arakawa formula

Abstract: The cotangent zeta function is related to partial zeta functions of real quadratic fields and Hecke L-functions. Berndt and Arakawa explicitly evaluated its special values for special and general cases, respectively. We provide an elementary proof for this formula that does not use analytic continuation. We believe this method can also be applied to the evaluation of special values for other Dirichlet series. This is a joint work with Masaaki Furusawa.

Sep. 4 (Thu.)

9:20 – 10:10 **Miyu Suzuki** (Kyoto University)

Title: Distinguished discrete series representations

Abstract: Distinguished representations for groups over local fields are studied in relation to local Langlands correspondence. Recently, intertwining periods and their functional equations have been found to be useful for the study of distinguished representations. In this talk, we see some examples that intertwining periods are used to classify distinguished discrete series representations. This is joint work with Nadir Matringe.

10:30 – 11:20 **Yao Cheng** (Tamkang University)

Title: TBA

Abstract: TBA

11:40 – 12:30 **Nozomi Ito** (Kyoto University)

Title: Between $\mathfrak{z}$-finiteness and Hecke-finiteness

Abstract: Let G be a reductive group over a number field F. Fix a maximal compact subgroup K of $G(\mathbb{A}_F)$ and put $\mathfrak{z} = \mathrm{Center}(U(\mathrm{Lie}(G(\mathbb{R} \otimes_{\mathbb{Q}} F)) \otimes_{\mathbb{R}} \mathbb{C}))$. Let f be a $\mathbb{C}$-valued, smooth, and K-finite function on $G(F)\backslash G(\mathbb{A}_F)$ of uniform moderate growth. We consider the following two conditions.

(1) f is $\mathfrak{z}$-finite (i.e. f is an automorphic form).

(2) f is Hecke-finite.

As is well known, (1)⇒(2) is true (Harish-Chandra). Then, is (2)⇒(1) also true? If this is true, then we no longer need to worry about the $\mathfrak{z}$-finiteness when we study explicit constructions of functorial liftings.

In the first half of this talk, I will show (2)⇒(1) for classical groups. For practical purposes, this result is often sufficient. However, the proof I will show depends crucially on the endoscopic classification and is far from the proof of (1)⇒(2). Is there a more general proof? In the second half, I will discusses this question.

14:00 – 14:50 **Shu-Yen Pan** (National Tsing Hua University)

Title: On the Classification of Irreducible Representations of Finite Classical Groups

Abstract: Irreducible representations of finite reductive groups are classified by Lusztig in early 80s. In this talk, we would like to discuss the relation between Lusztig's classification and the following two (conjectural) classifications:

(1) a parametrization by rational conjugacy classes

(2) Vogan's conjecture on local Langlands correspondence

for finite symplectic or orthogonal groups.

15:10 – 16:00 **Hiroshi Ishimoto** (Osaka Metropolitan University)

Title: Arthur's classification for non-quasi-split odd special orthogonal groups

Abstract: In 2013, Arthur established a classification of discrete automorphic representations of quasi-split orthogonal and symplectic groups, which is called Arthur's classification. Following the work, Mok analogously proved it for quasi-split unitary groups. Later, Kaletha–Minguez–Shin–White gave the classification for non-quasi-split unitary groups, by using the result of Mok. Now we can analogously prove it for non-quasi-split odd special orthogonal groups by the method of Kaletha–Minguez–Shin–White, and by using the result of Arthur. In this talk, I would like to explain the statement of Arthur's classification for non-quasi-split odd special orthogonal groups, and the difference of the proof from the case of unitary groups.

16:20 – 17:10 **Masao Oi** (National Taiwan University)

Title: Green functions for positive-depth Deligne–Lusztig induction

Abstract: In 1976, Deligne and Lusztig introduced a certain algebraic variety (called a Deligne–Lusztig variety) and proved that any irreducible representation of a finite group of Lie type appears in the alternating sum of its l-adic cohomology (called a Deligne–Lusztig representation). One of the crucial features of their theory is an explicit character formula of Deligne–Lusztig representations, which is expressed in terms of "Green functions". Recently, Chan and Ivanov introduced a p-adic version of a Deligne–Lusztig variety, leading to a p-adic version of a Deligne–Lusztig representation. In this talk, I would like to explain that we can define a "Green function" also in this context and establish an explicit character formula using it. I am also planning to explain its applications, including an exhaustion result for a certain class of irreducible representations of p-adic reductive groups. This is joint work with Charlotte Chan (University of Michigan).

Sep. 5 (Fri.)

9:20 – 10:10 **Satoshi Kumabe** (Kyushu University)

Title: Zeta functions of certain two-parameter families of K3 surfaces via the Appell functions

Abstract: The hypergeometric functions are given as complex power series with parameters. On the other hand, analogous functions on finite fields, called hypergeometric functions over finite fields, are defined as functions on finite fields with multiplicative characters as parameters. In this talk, we apply the hypergeometric functions over finite fields to describe the zeta functions of certain 2-parameter families of $K3$ surfaces over finite fields. In particular, we use the Appell functions, which are hypergeometric functions of two variables, over finite fields to connect the zeta functions of these $K3$ surfaces to those of the Legendre elliptic curves.

10:30 – 11:20 **Shii Ryota** (Kyushu University)

Title: On the Mazur–Tate refined conjecture for the anticyclotomic tower at inert primes

Abstract: B. Mazur and J. Tate defined elements of a group ring called Mazur–Tate elements that interpolate the special values of the Hasse–Weil L-function of an elliptic curve E over $\mathbb{Q}$. They also formulated a conjecture regarding the relation between these elements and the Fitting ideal of the Selmer group of E over a cyclotomic extension of $\mathbb{Q}$. When we consider, as a base field, a subfield of the anticyclotomic $\mathbb{Z}_p$-extension of an imaginary quadratic field K, it depends deeply on the behavior of the prime number p in $K/\mathbb{Q}$. When p splits in $K/\mathbb{Q}$, there are works by C.-H. Kim in the cases both when E has a good ordinary and supersingular reduction at p, using Bertolini–Darmon elements. We obtain a similar result for the case where p is inert in $K/\mathbb{Q}$ and E has a good supersingular reduction at p. In this talk, we introduce the main result and its proof.

11:40 – 12:30 **Hohto Bekki** (Saga University)

Title: Regulators and L-values of some Fermat hypersurfaces

Abstract: This is joint work in progress with Lambert A'Campo and Aleksander Horawa. The Beilinson conjecture predicts that (the transcendental part of) special values of L-functions of motives are described by their regulators. Motivated by the work of Otsubo, who gave an explicit computation of the regulators of Fermat curves, we are trying to find some explicit formulas for the regulators of higher-dimensional Fermat hypersurfaces. In this talk, I would like to report on some curious integral representations of $\zeta(3)$, the Catalan constant, an L-value of a certain Asai L-function of a Hilbert modular form, etc., that we observed in the course of this project.